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NONLINEAR PROBLEM OF A CIRCULAR CYLINDER RISING VERTICALLY TO AN INTERFACE BETWEEN LIQUID MEDIA

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The nonlinear initial-boundary-value problem of a contour approaching an interface between two liquid media is considered. A solution is constructed using a previously developed numerical method that is based on reducing the original problem to a system of integrodifferential equations for singularities simulating liquid and rigid boundaries and a function that describes the interface between the media. Calculation results for the perturbations generated by a circular cylinder approaching a free surface are presented. The dependences of the flows obtained and the hydrodynamic characteristics of the contour on the Froude number are estimated.

Solving nonlinear unsteady problems of the motion of a contour near interfaces between media has become possible owing to the development of computational hydrodynamics. In this area, the problem of a contour rising vertically to an interface between media is a problem of special interest, which has broad practical applications. Two-dimensional potential flow about a circular cylinder approaching a free surface was considered in [1]. The cylinder in the state of deep submersion is gradually accelerated from zero to constant vertical velocity. The generalized vortex method developed in [2, 3] was used to obtain the elevation of the free surface and the streamlines. Calculation results, including the pressure distribution along the contour, were presented for several values of the velocity. The results were compared with calculations for a cylinder approaching a rigid wall and a cylinder moving in an unbounded fluid. In [4], the boundary-element method was used for calculation of the vertical (up and down) motion of a cylinder under a free surface. Passing through the free surface, the body carries a fluid layer. This phenomenon is explained by the presence of inertial forces. The two-dimensional nonlinear unsteady problem of surface waves generated by vertical motion of a cylinder was considered in [5]. The initial stage of the process was studied. The method of solution implies that the condition of no separation of the flow about the cylinder is strictly satisfied. The velocity potential, the displacement of the free surface, and the coordinates of points of the cylinder were presented as power series in time. The shape of the free surface at the initial stage of the process was calculated for a cylinder moving upward to the free surface. The horizontal motion of a cylinder and the motion at an angle to the horizon were also studied. The same problem was considered in [6] using a dipole approximation. The motion of a circular cylinder at constant velocity to a free surface of a heavy fluid was studied in [7, 8]. Calculations of the free-surface shape were compared with the data obtained by asymptotic expansions in [5]. The method proposed in this work makes it possible to calculate flows at any time before water exit of the body. We should note that this problem was considered previously [9] using the complex boundary element method proposed in [10]. The process in which a body approaches a free surface at a short distance was studied. In [11], the water exit of a body was investigated for large Froude numbers. The finite volume-method was used for discretization of Navier–Stokes equations. The deformations of the free surface produced by a circular cylinder were calculated using a multilevel grid scheme. Calculations of

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the free-surface shape and flow about the cylinder at various times were given.

The aim of the present paper is to study the problem of a circular cylinder rising vertically to a free surface using the numerical method developed previously. Particular attention is given to evaluation of the effect of the Froude number on the flow about the cylinder.

We consider the problem of a contour L_0 which rises vertically to the interface L_1. In the semi-infinite layers D_1 and D_2 (D_1 is the lower layer), the fluid is ideal, incompressible, heavy, and homogeneous. The coordinate system is chosen in such a manner that the x axis coincides with interface L_1, which is not perturbed at the initial time. We introduce the following notation: g is the acceleration of gravity, ρ_k is the density of the layer D_k, and R is the radius of the cylinder.

We shall use singularities to model the liquid and rigid boundaries. To this end, we consider a vortex sheet with intensity $\gamma_1(s_1,t)$ along the interface $L_1(t)$ and a layer of sources with intensity $q(s_0,t)$ along the contour $L_0(t)$. We assume that $\gamma_1(\pm\infty,t)=0$. In the domains $D_1(t)$ and $D_2(t)$, the fluid motion is described by the function

$$\bar{V}(z,t) = \frac{1}{2\pi i}\int\limits_{L_1(t)} \frac{\gamma_1(s_1,t)\,ds_1}{z-\zeta(s_1)} + \frac{1}{2\pi}\int\limits_{L_0(t)} \frac{q(s_0,t)\,ds_0}{z-\zeta(s_0)}. \tag{1}$$

The system of integrodifferential equations corresponding to the kinematic and dynamic boundary conditions at the interface $L_1(t)$ and the condition of no normal flow through the contour $L_0(t)$ has the form [12]

$$\frac{\partial z(s_1)}{\partial t} = V_1(z(s_1),t), \qquad z(s_1)\in L_1(t); \tag{2}$$

$$\frac{\partial G(s_1,t)}{\partial t} = \rho_*\Big(\frac{|\bar{V}_1(z(s_1),t)|^2}{2} - g\,\mathrm{Im}\,z(s_1) - \frac{\gamma_1^2(s_1,t)}{8}\Big), \quad z(s_1)\in L_1(t),$$

$$\rho_* = \frac{\rho_1-\rho_2}{\rho_1+\rho_2}, \qquad G(s_1,t) = \int\limits_{-\infty}^{s_1}\Big(\frac{\gamma_1(\sigma_1,t)}{2} + \rho_* V_{1s}(\sigma_1,t)\Big)\,d\sigma_1, \tag{3}$$

$$V_{js}(s_j,t) = \mathrm{Re}\,(V_j(z(s_j)),t)e^{i\theta_j(s_j,t)}), \qquad z(s_j)\in L_j(t) \quad (j=0,\,1);$$

$$\frac{q(s_0,t)}{2} = \mathrm{Im}\,((\bar{V}_0(z(s_0)),t) - \bar{V}_{L_0}(t))e^{i\theta_0(s_0,t)}), \qquad z(s_0)\in L_0(t); \tag{4}$$

$$V_{L_0}(t) = \begin{cases} iU_0\sin(\pi\tau/2), & 0\leqslant\tau\leqslant 1,\\ iU_0, & \tau>1. \end{cases} \tag{5}$$

Here $\bar{V}_j(z(s_j),t)$ is defined by formula (1) for $z(s_j)\in L_j(t)$ ($j=0,\,1$), $\theta_j(s_j,t)$ is the angle between the tangent at the point $z(s_j)\in L_j(t)$ and the x axis ($j=0,\,1$), $V_{L_0}(t)$ is the complex velocity corresponding to the gradual vertical acceleration of the circular cylinder from zero to a constant velocity, and $\tau = tU_0/R$ is the dimensionless time.

In the domains $D_1(t)$ and $D_2(t)$, the perturbations of the velocity and the interface damp at infinity:

$$\lim_{x\to\pm\infty}\bar{V}(z,t)=0, \qquad \lim_{x\to\pm\infty}\mathrm{Im}\,z(s_1)=0, \qquad z(s_1)\in L_1(t). \tag{6}$$

There are no perturbations of the velocity and the interface at the initial time:

$$\mathrm{Im}\,z(s_1)=0, \qquad z(s_1)\in L_1(0), \qquad \gamma_1(s_1,0)=q(s_0,0)=0. \tag{7}$$

Solving system (1)–(7), we determine the hydrodynamic pressure $p(s_0,t)$ at points of the contour $z(s_0)$ and the total hydrodynamic forces R_x and R_y:

$$p(s_0,t) - f(t) = -\rho_1\Big(\frac{\partial}{\partial t}\int\limits_0^{s_0} V_{0s}(\sigma_0,t)\,d\sigma_0 - \mathrm{Re}\Big(\bar{V}_{L_0}(t)V_0(z(s_0)),t)\Big) + \frac{|\bar{V}_0(z(s_0)),t)|^2}{2}\Big); \tag{8}$$

$$R_x - iR_y = i \int\limits_{L_0(t)} (p(s_0, t) - f(t))\mathrm{e}^{-i\theta_0(s_0,t)}\, ds_0. \tag{9}$$

Here $f(t)$ is a certain function of time only.

The system of integrodifferential equations (2)–(4) is nonlinear, and this is due to the following two factors: 1) the intensities of the singularities $\gamma_1(s_1, t)$ and $q(s_0, t)$ are included in a nonlinear fashion in the boundary conditions; 2) the shape of the interface $L_1(t)$ is unknown and is to be found by solving the system. This introduces certain difficulties into the solution of this system.

Equations (2) and (3) are integrated with respect to time using a Runge–Kutta–Felberg scheme of 5th order accuracy [13]. At each time t_n $(n = 1, 2, \ldots)$, we calculate the function $G^n(s_1^n)$ and the points on the interface $z^n(s_1^n) \in L_1^n$ (the superscript n refers to the function values at the nth time step). At each time step, determination of $\gamma_1^n(s_1^n)$ and $q^n(s_0)$ reduces to solving the following system of integral equations

$$\frac{\gamma_1^n(s_1^n)}{2} + \rho_* V_{1s}^n(s_1^n) = \frac{\partial G^n(s_1^n)}{\partial s_1^n}, \qquad z^n(s_1^n) \in L_1^n; \tag{10}$$

$$\frac{q^n(s_0)}{2} = \mathrm{Im}\,((\bar{V}_0^n(z^n(s_0)) - \bar{V}_{L_0}^n)\mathrm{e}^{i\theta_0^n(s_0)}), \qquad z^n(s_0) \in L_0^n. \tag{11}$$

The system of integral equations (10) and (11) was solved by the high-order panel method [14]. The contours L_1^n and L_0^n were divided into the intervals $[s_{1i-1}^n, s_{1i}^n]$ $(i = 1, \ldots, N)$ and $[s_{0j-1}, s_{0j}]$ $(j = 1, \ldots, M)$, respectively. The collocation points $z^n(s_{1i}^{n*}) \in L_1^n$ $(s_{1i}^{n*} \in [s_{1i-1}^n, s_{1i}^n])$ and $z^n(s_{0j}^*) \in L_0^n$ $(s_{0j}^* \in [s_{0j-1}, s_{0j}])$ were chosen in these intervals. Equations (10) and (11) were considered at the points $z^n(s_{1i}^{n*})$ $(i = 1, \ldots, N)$ and $z^n(s_{0j}^*)$ $(j = 1, \ldots, M)$. The interface L_1^n in the ith interval $[s_{1i-1}^n, s_{1i}^n]$ and the contour L_0^n in the jth interval $[s_{0j-1}, s_{0j}]$ were approximated by parabolas, and the unknown functions $\gamma_1^n(s_1^n)$ and $q^n(s_0)$ in the same intervals were approximated by linear functions. Discretization of the integral equations (10) and (11) and allowance for (1) leads to a system of linear algebraic equations for values of the functions $\gamma_1^n(s_1^n)$ and $q^n(s_0)$ at the ends of the intervals. After solving this system, from (1), we obtain the values of $\bar{V}^n(z)$ at points of the contour and from (8) and (9), we have the distributed and integral hydrodynamic characteristics.

Because of the symmetry of the flow domain relative to the y axis, the calculation domain was taken within the interval $0 \leqslant x/R \leqslant 10$. The numbers of points on the interface and the contour were 400 and 80, respectively. In the interval $7.5 \leqslant x/R \leqslant 10$, a damping layer was introduced using the technique described in [15] in order to suppress the waves reflected from the boundaries of the calculation domain. Kelvin–Helmholtz instability was prevented using the filtration procedure proposed in [16]. The derivative $\partial G(s_1, t)/\partial s_1$ in (10) and the contour integral in the expression for the total hydrodynamical forces (9) were calculated using cubic splines. We should note that the procedures and techniques described allowed us to carry out calculations over wide ranges of the density ratio ρ_* and the Froude number $\mathrm{Fr} = U_0/\sqrt{gR}$, and with a change in the total energy E not exceeding 0.12%.

Using a more general method developed to solve the problem of the vertical upward motion of a cylinder to an interface between two media, we considered the case of cylinder approaching a free surface of a homogeneous fluid $(\rho_* = 1)$, which has broad practical applications. At the initial time, the center of the cylinder is located at the point with the coordinates $(x_c(0), y_c(0)) = (0, -5)$, and it begins to rise gradually according to the law (5). Results of the numerical simulation of the effect of the Froude number on the hydrodynamic forces exerted on the cylinder and on the free-surface perturbation are shown in Figs. 1–3.

The calculation was performed up to the time τ_* at which the strong interaction between the cylinder and the free surface led to divergence of the numerical method. Table 1 gives the limiting values of time τ_* for various Froude numbers Fr. For comparison, the values of τ_* obtained in [1] are $\tau_* = 4.24$, 4.52, and 5.80 for $\mathrm{Fr} = 0.2$, 0.4472, 1.4142, respectively. As follows from the results presented herein, the proposed method makes it possible to obtain a considerable increase in τ_* and to study the most interesting regimes of interaction of the contour with the free surface.

Calculations of the free-surface shape at various times are shown in Fig. 1. Passing through the unperturbed level of the free surface, the body carries a fluid layer because of the presence of inertial forces.

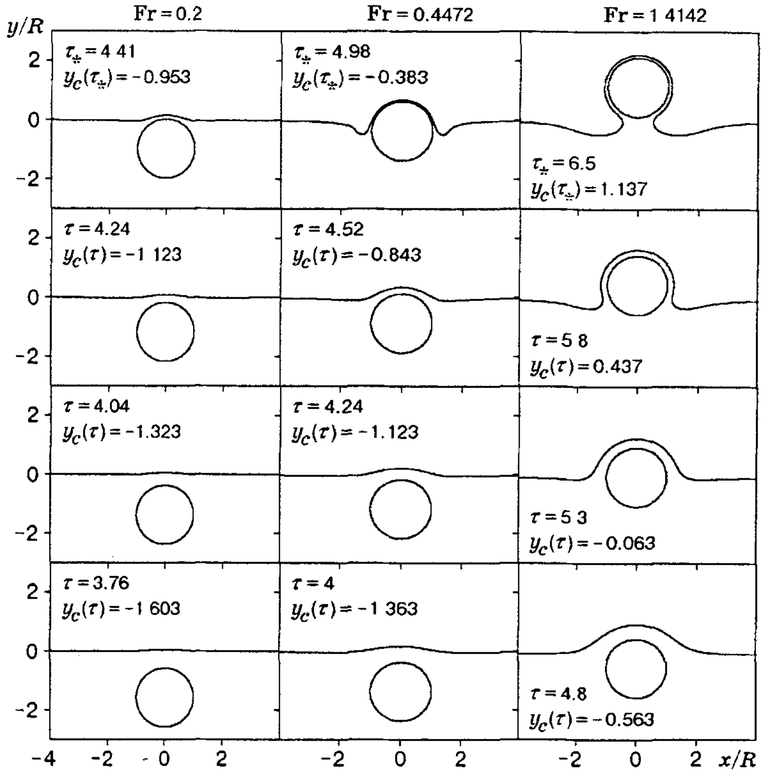

Fig. 1. Free-surface perturbations produced by a circular cylinder rising vertically.

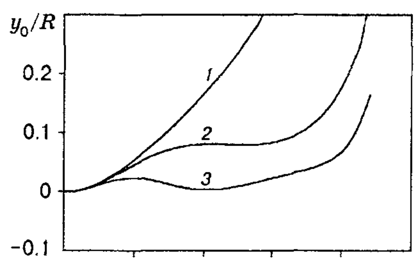

Fig. 2

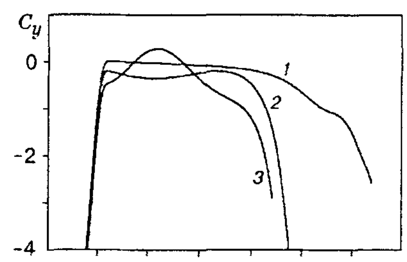

Fig. 3

Fig. 2. Free-surface elevation at the point located directly above the center of the cylinder for Fr = 1.4142 (1), 0.4472 (2), and 0.2 (3).

Fig. 3. Hydrodynamic forces exerted on the cylinder rising vertically to the free surface for Fr = 1.4142, 0.4472 (2), and 0.2 (3).

TABLE 1

Fr	τ_*	Fr	τ_*
0.2	4.41	0.9	5.57
0.3	4.62	1.0	5.67
0.4	4.87	1.1	5.78
0.4472	4.98	1.2	5.89
0.5	5.07	1.3	6.11
0.6	5.25	1.4	6.42
0.7	5.36	1.4142	6.50
0.8	5.45	1.5	6.69

Thus, at large Froude numbers, it is possible to calculate the flow about the cylinder which is above the unperturbed level of the free surface, and this is observed at Fr = 1.4142. A similar flow pattern for large Fr was found experimentally in [17]. We note that the results for the free-surface shape obtained here are in satisfactory agreement with the data given in [1].

The behavior of the free surface at Fr = 0.2 has an interesting feature. During acceleration up to time $\tau = 1$, an increase in the free-surface elevation at the point with the ordinate y_0 is observed. After that, the elevation decreases until a certain moment and then grows again (see Fig. 2). At the same time, for Fr = 0.4472 and 1.4142 the free surface at this point increases during the entire motion of the cylinder.

The hydrodynamic force $C_y = 2R_y/(\rho_1 R U_0^2)$ for the same Froude numbers is shown in Fig. 3. We should note that there is an interval of positive values of C_y for Fr = 0.2 (due to buoyancy force) and the resistance increases sharply for all Froude numbers when the limiting times τ_* are approached.

Thus, using the method developed previously, we solved the nonlinear problem of a circular cylinder rising vertically to a free surface: Comparison with known results shows that this method has a number of advantages. In particular, it can be used to study the regimes of strong interaction between the contour and the free surface in detail.

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REFERENCES

1. J. G. Telste, "Inviscid flow about a cylinder rising to a free surface," *J. Fluid Mech.*, **182**, 149–168 (1987).
2. G. R. Baker, D. I. Meiron, and S. A. Orszag, "Generalized vortex methods for free-surface flow problems," *J. Fluid Mech.*, **123**, 477–501 (1982).
3. G. R. Baker, D. I. Meiron, and S. A. Orszag, "Application of a generalized vortex methods to nonlinear free-surface flows," in: *Proc. of the 3rd Int. Conf. on Numerical Ship Hydrodynamics*, Paris (1981), pp. 179–191.
4. A. G. Terentiev, K. E. Afanasiev, and M. M. Afanasieva, "Simulation of unsteady free surface flow problems by the direct boundary element method," in: *Advanced Boundary Elements Methods:* Proc. of the IUTAM Symp., San Antonio (1988), pp. 427–434.
5. P. A. Tyvand and T. Miloh, "Free-surface flow due to impulsive motion of a submerged circular cylinder," *J. Fluid Mech.*, **286**, 67–101 (1995).
6. P. A. Tyvand and T. Miloh, "Free-surface flow generated by a small submerged circular cylinder starting from rest," *ibid.*, 103–116 (1995).
7. M. Greenhow and S. Moyo, "Water entry and exit of horizontal circular cylinders," *Philos. Trans. Roy. Soc. London*, **A355**, 551–563 (1997).

8. S. Moyo, "Hydrodynamic interaction of horizontal circular cylinders with a free surface," Ph.D. Thesis, Univ. of Brunel (1997).
9. M. Greenhow, "Water entry and exit of horizontal circular cylinder," *Appl. Ocean Res.*, **10**, No. 4, 191–198 (1988).
10. T. Vinje and P. Brevig, "Nonlinear ship motions," in: *Proc. of the 3rd Int. Conf. on Numerical Ship Hydrodynamics*, Paris (1981), pp. 257–266.
11. I. R. Park and H. H. Chun, "Analysis of flow around a rigid body in water-entry and exit problems by finite volume method," in: *Proc. of the 6th Symp. on Nonlinear and Free-Surface Flows*, Hiroshima (1998), pp. 13–18.
12. S. I. Gorlov, "Unsteady nonlinear problem of the horizontal motion of a contour under an interface between two liquid media," *Prikl. Mekh. Tekh. Fiz.*, **40**, No. 3, 37–43 (1999).
13. G. E. Forsythe, M. M. Malcolm, and C. B. Moler, *Computer Methods of Mathematical Computations*, Prentice-Hall, Englewood Cliffs, New Jersey (1977).
14. J. L. Hess, "Higher-order numerical solution of the integral equation for the two-dimensional Neumann problem," *Comput. Methods Appl. Mech. Eng.*, **2**, No. 1, 1–15 (1973).
15. M. Israeli and S. A. Orszag, "Approximation of radiation boundary conditions," *J. Comput. Phys.*, **41**, 115–135 (1981).
16. M. S. Longuet-Higgins and E. D. Cokelet, "The deformation of steep surface waves on water. I. A numerical method of computations," *Proc. Roy. Soc. London*, **A350**, 1–26 (1976).
17. O. M. Faltinsen, *Sea Loads on Ships and Offshore Structures*, Cambridge Univ. Press, Cambridge (1990).